Methodology of Mathematical Modeling

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Where innovation starts

Mathematics for Industry

Mathematical Models

- **Model:**
  - simplified representation of certain aspects of a real system
  - capturing the essence of that system

- **Mathematical Model:**
  - using mathematical concepts:
    - variables
    - operators
    - functions
    - equations
    - inequalities

Mathematical Models

- **First-Principle Models**

  based on physical laws (e.g. Newton's second law)

  descriptive, explaining

  material parameter values often not known (measurements)

- **Stochastic Models**

  based on distributions, averages (e.g. risk models)

  capable to deal with random phenomena,

  hard to distinguish relations

- **Empirical/data Models**

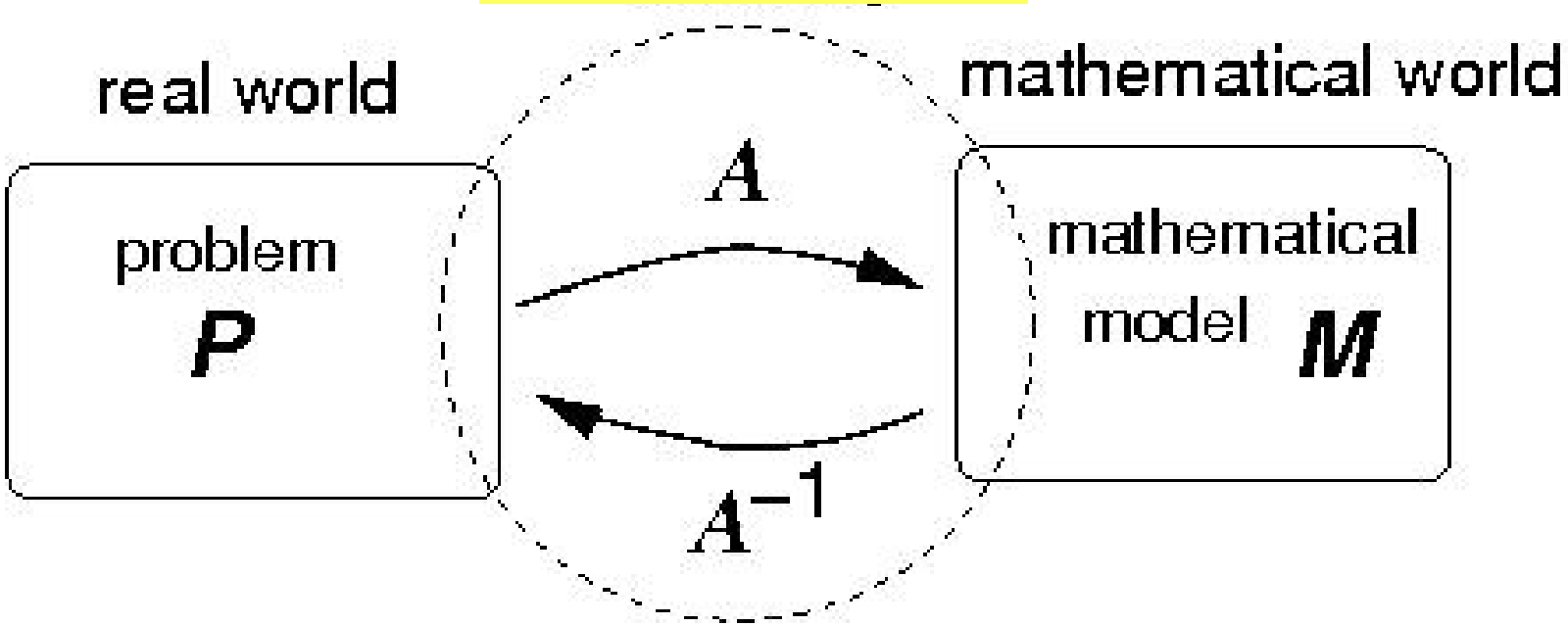
  based on (historical) patterns, data (e.g. Moore's law)

  not explaining, relations based on reality

Mathematical Modeling

- Metaphor
- Non-uniqueness
- Outcome
- Skills

Mathematical Modeling



Metaphor

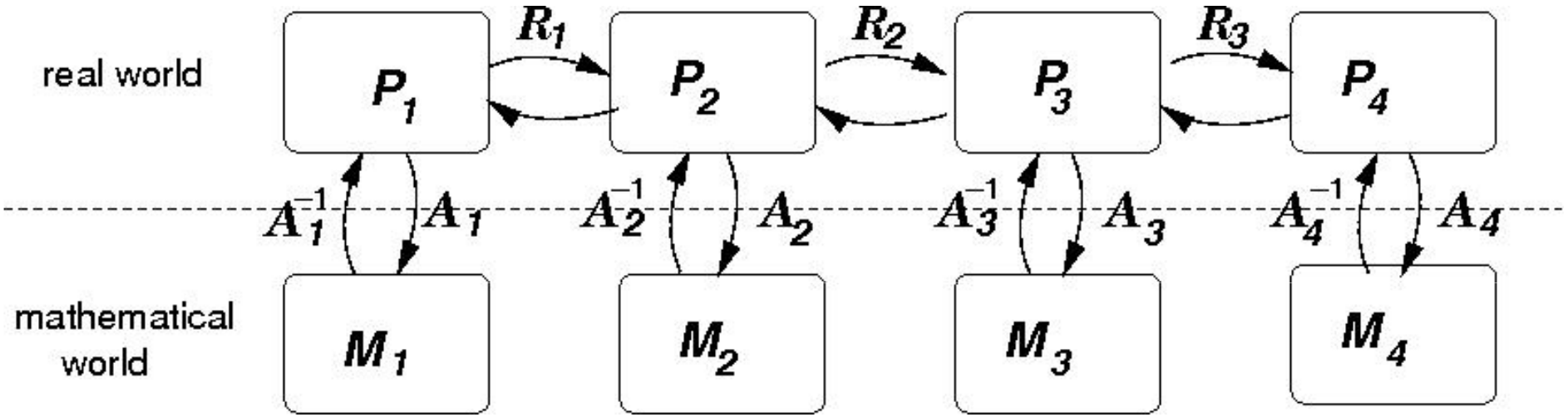
- **Forming of mental image of situation being modeled**
- **Symbolizes the problem**
- **Using concepts & analogies**

Creativity

Simplification

Non-uniqueness

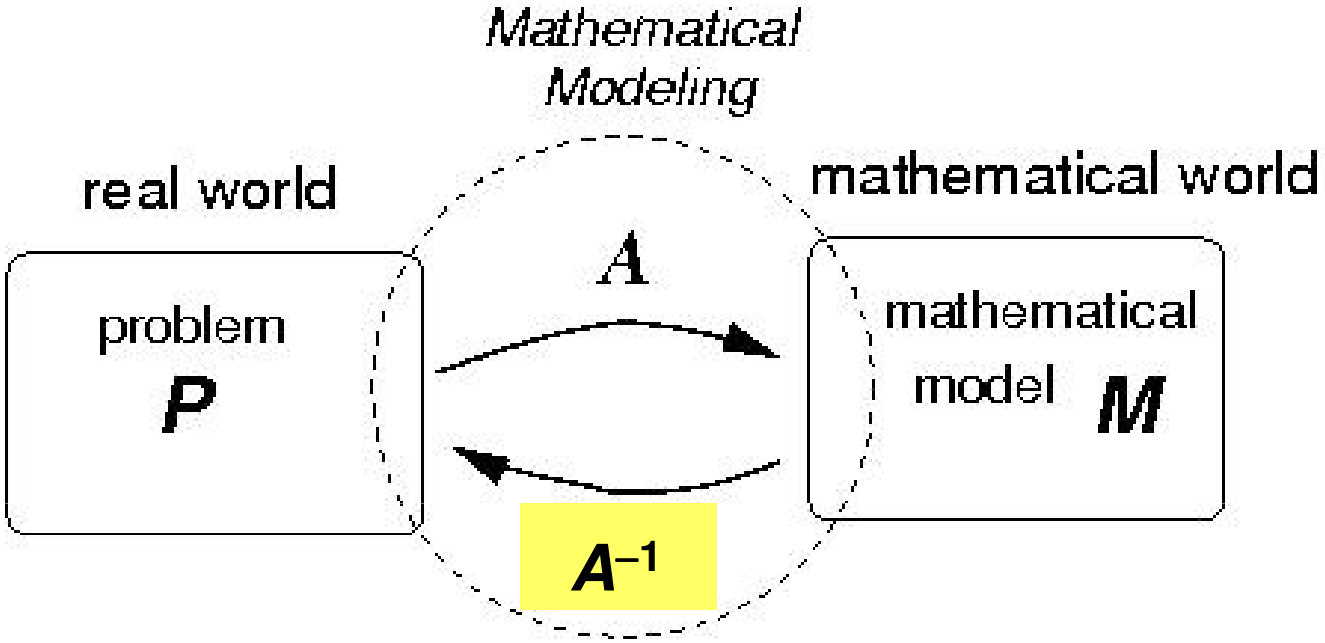
Simplification:



Non-uniqueness: choice of model depends on

- the form the solution needs to be (problem)
  accuracy
- availability of data

Outcome



Back to real world:

Recommendations

based on

- *qualitative* results
- *quantitative* results

Skills

- **Mathematics**

- **Creativity**

- **Conceptual thinking**

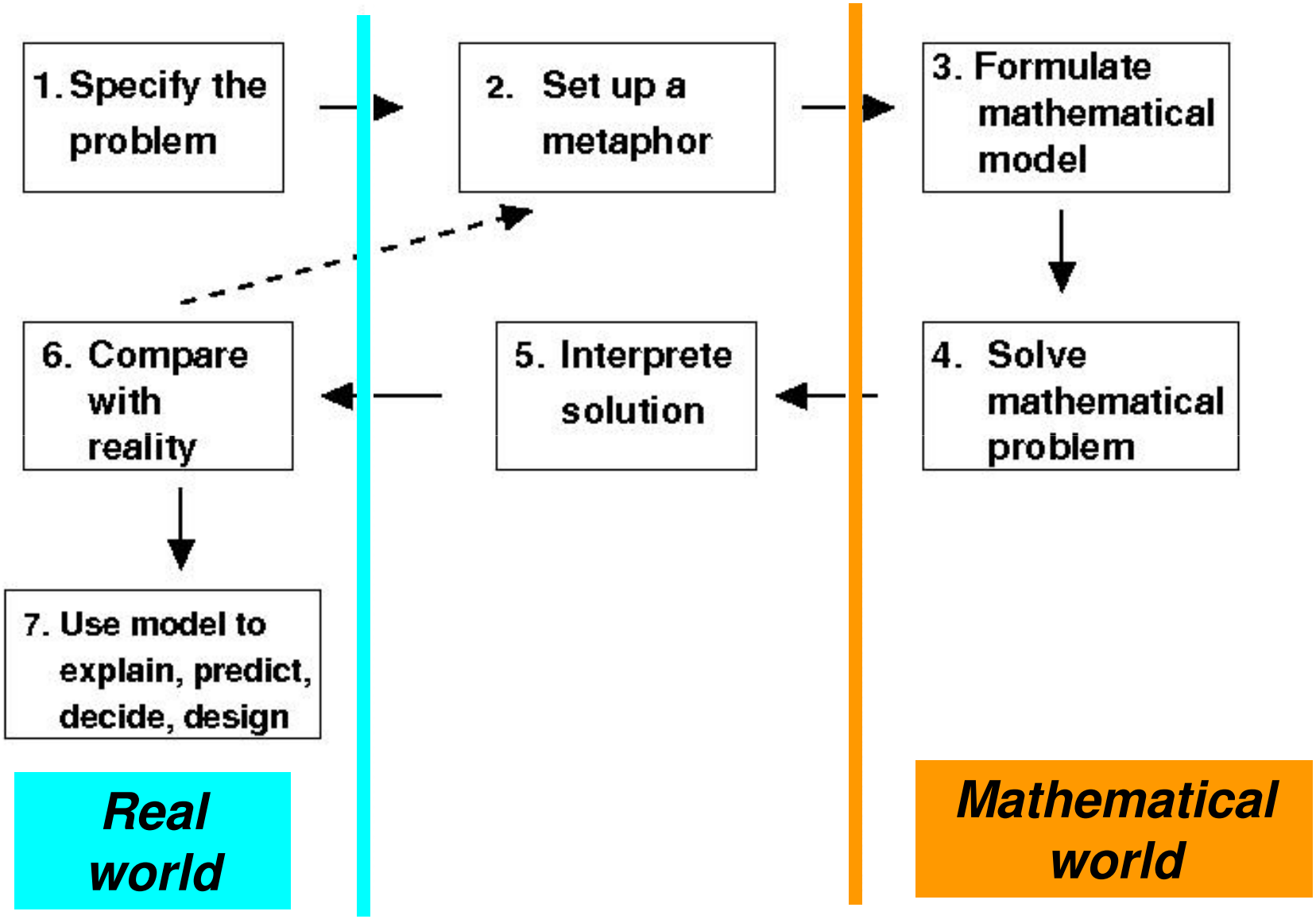
- **Communication skills**

Aims

- Investigate behaviour and relation of elements of problem
- Consider all possibilities, evaluate alternatives, exclude impossibilities
- Verification against measurements; result to be used in other operating regimes

- Optimization
- Facilitate design and proto-typing
- Substantiate decisions

Mathematical Modeling Cycle

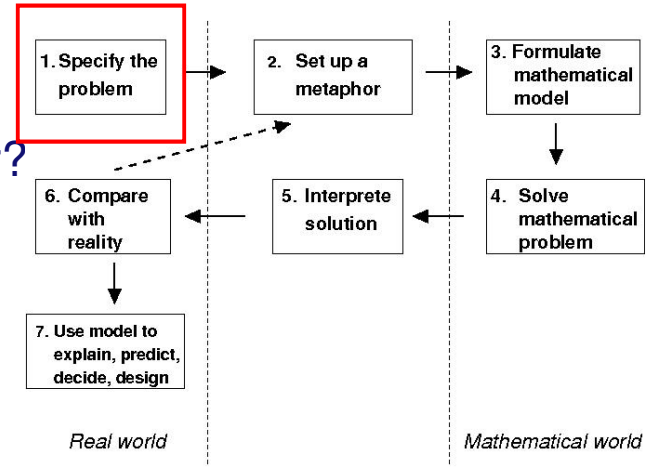


Step 1: Specify the Problem

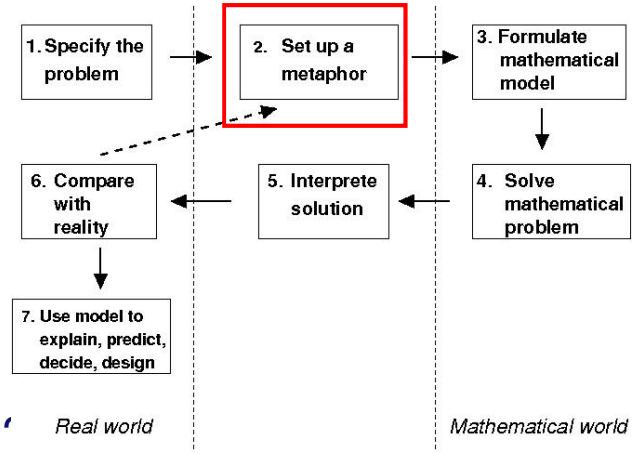
- What is background of the problem, its history and its causes.
- What do we want to know?
- Why is one interested in solving the problem? What are the limitations of the current practice?
- What is the solution needed for? What is the purpose of solving the problem?
- What are the constraints to solving the problem?
- Who needs a solution?
- What is the impact/benefit of solving the problem?
- What problem is solved if one has found an answer?
- How will the outcome be judged?
- What form does the solution need to have?

  What do you communicate to the Problem Owner?
- How would you implement the solution?
- Are there other related problems?
- What are the sources of facts and data,

  and are they reliable?



Step 2: Set up a metaphor



- What are the operative processes at work?
- What needs to be determined for solving the problem‘
- What are the main features, which ones are relevant and which ones are not?
- What is the relation between the main features?
- Which features do change?
- What is controllable in the problem?
- What are the conditions?
- What are the relevant timescales and dimensions of the problem?
- What kind of material is considered, and what is characteristic of that material?

Simplification ⇒ **Assumptions**

Step 2: Set up a metaphor

Assumptions ***made in real world***

- **whether or not to include certain features,** **and why**
- **about the relationships between features,**
- **about their relative effects.**

Decisions: ***In collaboration with Problem Owner***

- **level of detail,**

Step 3: Formulate Mathematical Model

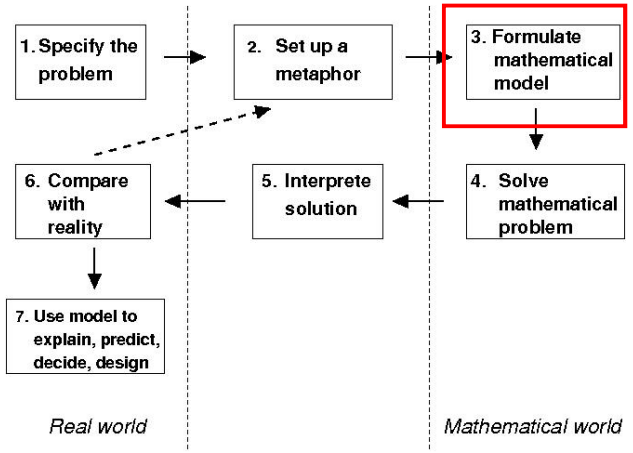
Translate relationships of metaphor

into

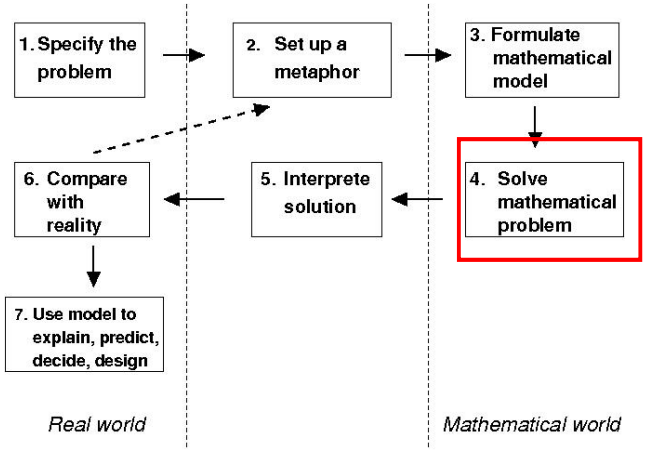
mathematical terms

Mathematical Model:

- **Input and Output**
- **Constants**
- **State variables**
- **Independent variables**
- **Domain**
- **(Boundary / initial / constraint) conditions**



Step 4: Solve Mathematical Model

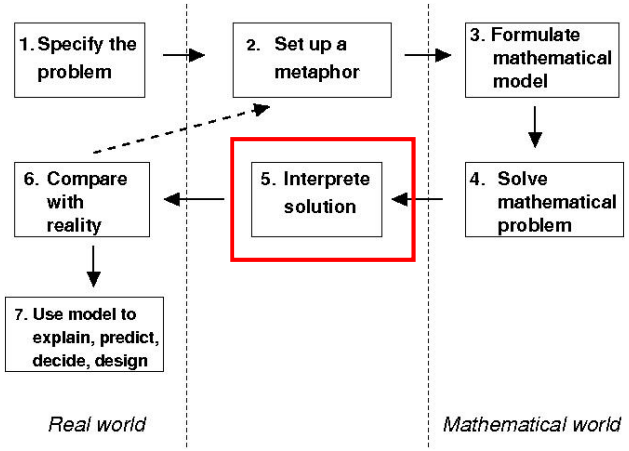


- **Analytically**
- **Numerically**

  make equations dimensionless

  calculations with O(1)-numbers

Step 5: Interprete Solution



Retraction of conceptual leap

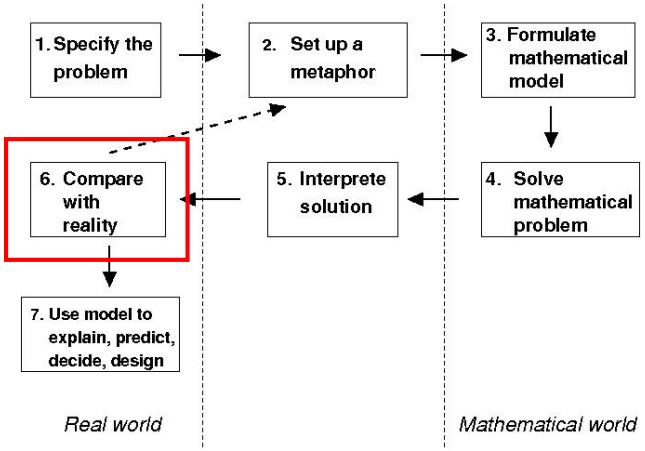
from

mathematical world

to

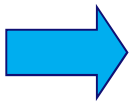
real-world problem

Step 6: Compare with Reality



- **Validation of model**
- **Extraction of model parameters**
- **Extension to other operating regimes**

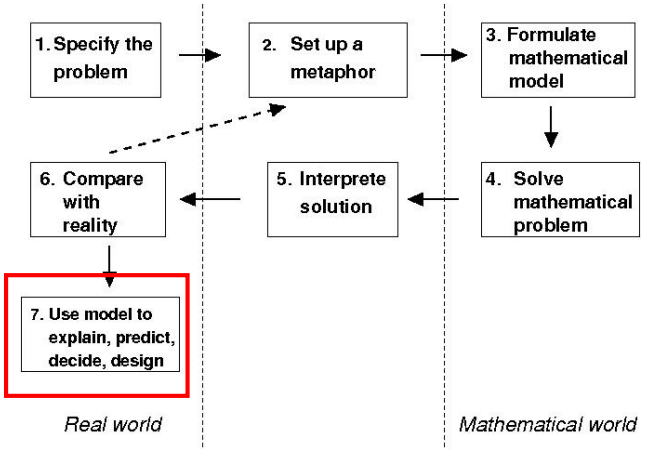
Model insufficient for solving problem



Re-enter modeling cycle again

Accuracy of measurements

Step 7: Use Model to Explain, Predict, Decide, Desig



- Determine:
  - typical behaviour
  - critical parameters
  - trends
  - dependency on control parameters

➡ ***Recommendations***

Summary

Mathematical Modeling

- driven by the problem from the real world;
- also includes the interpretation of the solution

Finding an appropriate model is an art:

Mathematical Modeling:

- *conceptual thinking*
- *appropriate simplifications*
- *close collaboration between Problem Owner and Mathematician.*

Literature

D. Edwards and M. Hamson, *Guide to Mathematical Modelling, MacMillan, 1989.*

J.S. Berry, D.N. Burghes, I.D. Huntley, D.J.G. James, and A.O.Moscardini (Eds.), *Teaching and Applying Mathematical Modelling, Wiley, 1984.*

T.L. Saaty and J.M. Alexander, *Thinking with Models, Pergamon Press, 1981.*

This course

Focus on:

Step 2: Set up a metaphor (mental image)

Step 3: Formulate mathematical model

Step 4: Solve mathematical problem

Step 5: Interprete solution

Through:

Theory Mathematical Physics / lectures

Problems / assignments